General Multilevel-Categoric Factorial Tutorial

Part 1 – Categoric Treatment

Introduction – A Case Study on Battery Life

Design-Expert® software version 10 offers a "Multilevel Categoric" option, also known as a "general factorial" on the "Factorial" design tab. If you have completed the General One-Factor Multilevel-Categoric Tutorial (recommended), you've seen how this option handles one multilevel, categorical factor. In this two-part tutorial you will learn how to set up a design for multiple categorical factors. Part 2 shows you how to convert truly continuous factors, such as temperature, from categorical to numerical. With this you can generate response surface graphs that provide a better perspective of your system.

If you are in a hurry, skip the boxed bits—these are sidebars for those who want to spend more time and explore things.

The experiment in this case, which comes from Montgomery's *Design and Analysis of Experiments*, seeks consistently long life in a battery that will be subjected to extremes in ambient conditions. It evaluates three materials (factor A) at three levels of temperature (factor B). Four batteries are tested at each of the nine two-factor combinations in a completely randomized design. The responses from the resulting 36 runs are shown below.

Material Type	Temperature (deg F)					
	15		70		125	
A1	130	155	34	40	20	70
	74	180	80	75	82	58
A2	150	188	136	122	25	70
	159	126	106	115	58	45
A3	138	110	174	120	96	104
	168	160	150	139	82	60

General factorial design on battery life (data in units of hours)

The following questions must be answered:

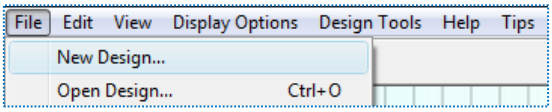
- How does material type and temperature affect battery life?
- Do any materials provide uniformly long life regardless of temperature?

The big payoff comes if the battery can be made more tolerant to temperature variations in the field.

This case study provides a good example of applying statistical DOE for robust product design. Let's get started on it!

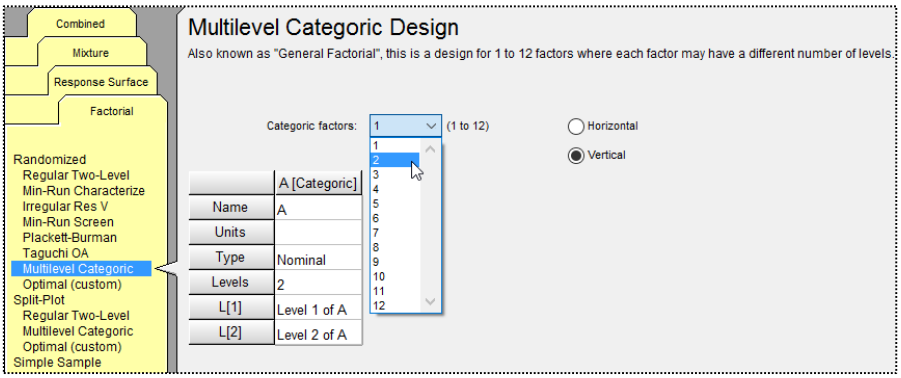
Design the Experiment

To build the design, choose **File**, **New Design** as shown below (or to save strokes, simply click the blank-sheet icon (🗋) on the toolbar).



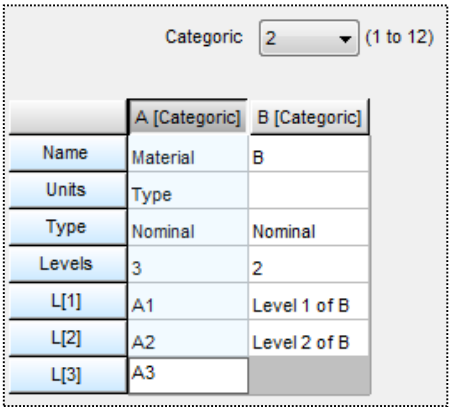
Starting a new design via menu (option: click blank-sheet icon (🗋) on the toolbar)

Then from the default **Factorial** tab, click **Multilevel Categoric**. Choose **2** as the number of factors. If you are in Horizontal entry mode, change it to **Vertical**. (Design-Expert will remember this the next time you set up a design.)



Selecting number of factors for multilevel-categoric general-factorial design

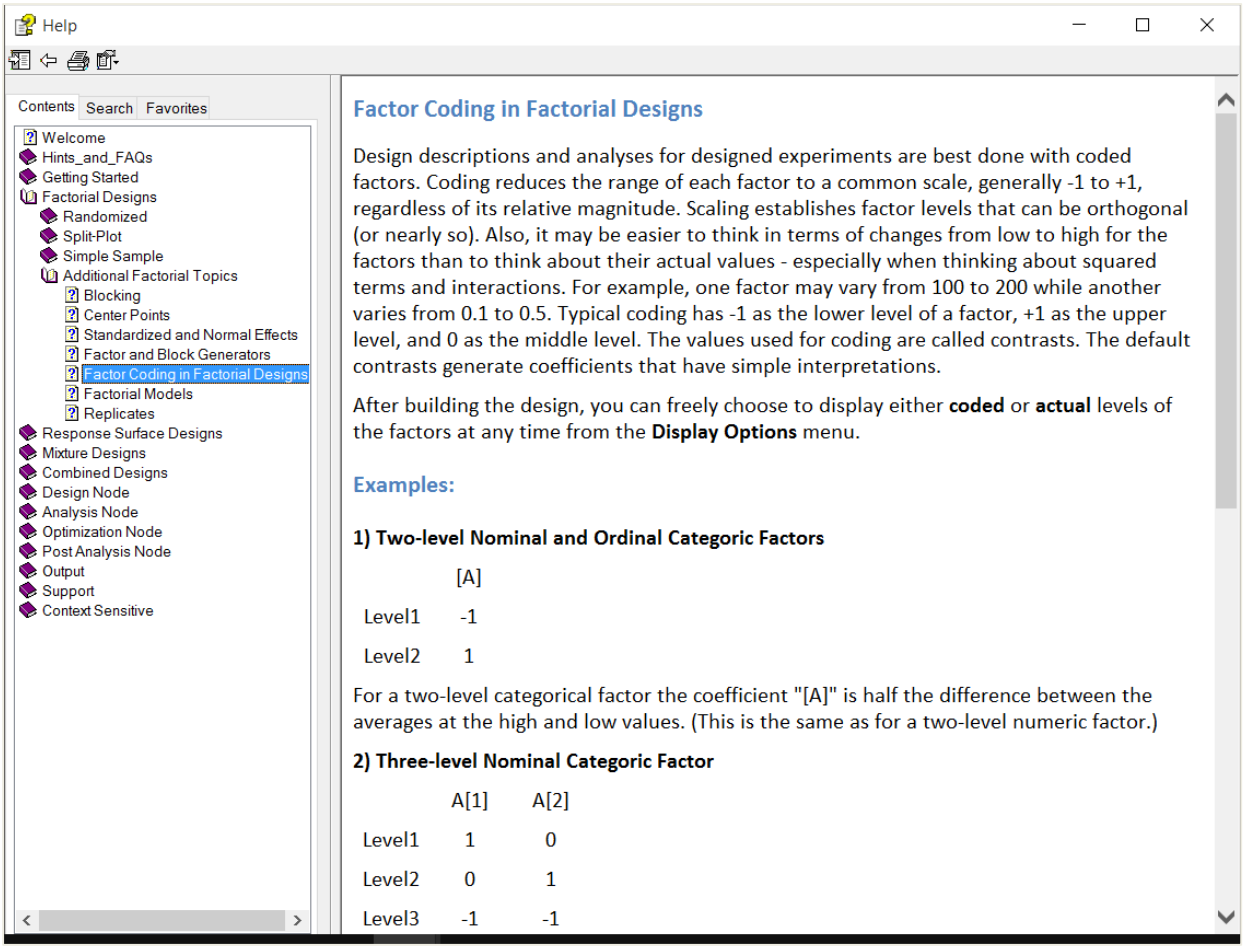
Enter **Material** for factor name A (Categoric). Key in the word **Type** as your **Units**. Enter the value **3** for the number of levels. Change the treatment names to **A1**, **A2** and **A3**. Notice that **Type** in the far left column defaults to **Nominal** (named) as opposed to ordinal (ordered). This difference in the nature of factors affects how Design-Expert codes the categorical levels, which changes the model coefficients reported under ANOVA in the subsequent response analysis. Your design should now appear as that shown below.



Entering material as a nominal factor

☒ ***Explore details in Help on Factor Types***: Tutorials such as this one on general factorials will quickly get you up to speed on how to use Design-Expert software, but it does not serve as a statistical primer for design and analysis of

experiments. If you crave such details, Help is at your fingertips! For example, go to **Help**, **Contents** and work your way down the tree structure through the branches from **Factorial Designs->Additional Factorial Topics->Factor Coding in Factorial Designs**. Note the details on the distinction in categoric contrasts (Nominal vs Ordinal).



Help on Factor Coding in Factorial Designs

Close Help by pressing **X** on its window.

Now enter factor B data by keying in **Temperature** for factor name B (Categoric), **deg F** for units, **3** for the number of levels, and **15**, **70** and **125** for the levels. Press **Nominal**, click the arrow on the drop list, then choose **Ordinal** as shown below. This change from **Nominal** to **Ordinal** indicates that although this factor is being treated categorically (for example, due to controls offering only the three levels), temperature is really a continuous factor. Click **Continue** at the screen's far lower right.

	A [Categoric]	B [Categoric]
Name	Material	Temperature
Units	Type	deg F
Type	Nominal	Nominal
Levels	3	Nominal / Ordinal
L[1]	A1	15
L[2]	A2	70
L[3]	A3	125

Entering information on factor B

Enter **4** for replicates. The number of runs (36) won't be updated until you press the Tab key or move from the cell. Leave the blocks option alone because these experiments are completely randomized.



Entering the number of replicates

Click **Continue** to move on to the entry screen for responses. Leave the default responses at 1. Enter name as **Life** and units as **hours**.

Now we will walk you through a calculation of power – the ability of your experiment to detect meaningful differences in treatments. If you do too few runs and under-power your experiment, an important change in response (the "signal") will become obscured by normal system/test variation (the "noise"). That would be a waste of time and materials. Design-Expert makes the calculation of power easy and puts it upfront in the design-building process so you have a chance to bolster your experiment, if necessary. Let's assume that battery life must improve by at least **50** hours to be of any interest and that quality control records produce a standard deviation of **30**. Enter these values as shown below, **Tab** (or click) out of **30,** and Design-Expert then calculates the signal to noise ratio.

Responses: 1 (1 to 999) Edit model for power...

	Name	Units	Diff. to detect Delta("Signal")	Est. Std. Dev. Sigma("Noise")	Delta/Sigma (Signal/Noise Ratio)
	Life	hours	50	30	1.66667

Response entry screen

Press **Continue** to see the power of this design for the difference that the engineers hope to detect, at a minimum. It is calculated to be 94.5 % probability of seeing a difference (delta) as small as 50 hours. This exceeds the rule-of-thumb for power of 80 % at a minimum, thus it can be concluded that the planned design will suffice.

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio.

Recommended power is at least 80%.

Life	hours		
Signal (delta) = 50.00		Noise (sigma) = 30.00	Signal/Noise (delta/sigma) = 1.67
A	B		
94.5 %	94.5 %		

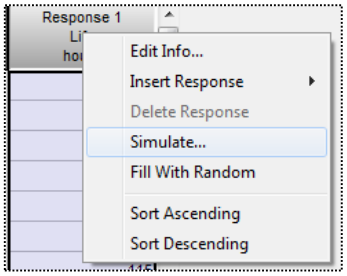
Power calculation

Click **Continue** to complete the design specification process. Design-Expert now displays the 36 runs (in random order) from the replicated 3x3 factorial design.

Analyze the Results

To save time, simulate the experimental results by right-clicking the response header and selecting **Simulate**.

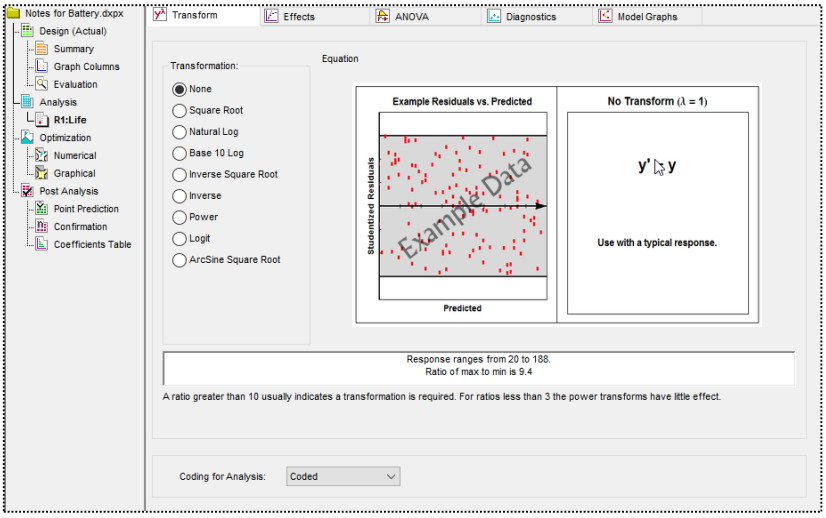
☒ ***Explore simulation tools***: A heads-up for statistics educators: You can build your own simulations via the Design Tools. Feel free to bring up the controls for this and press Help for details on using it.



Choosing a simulation

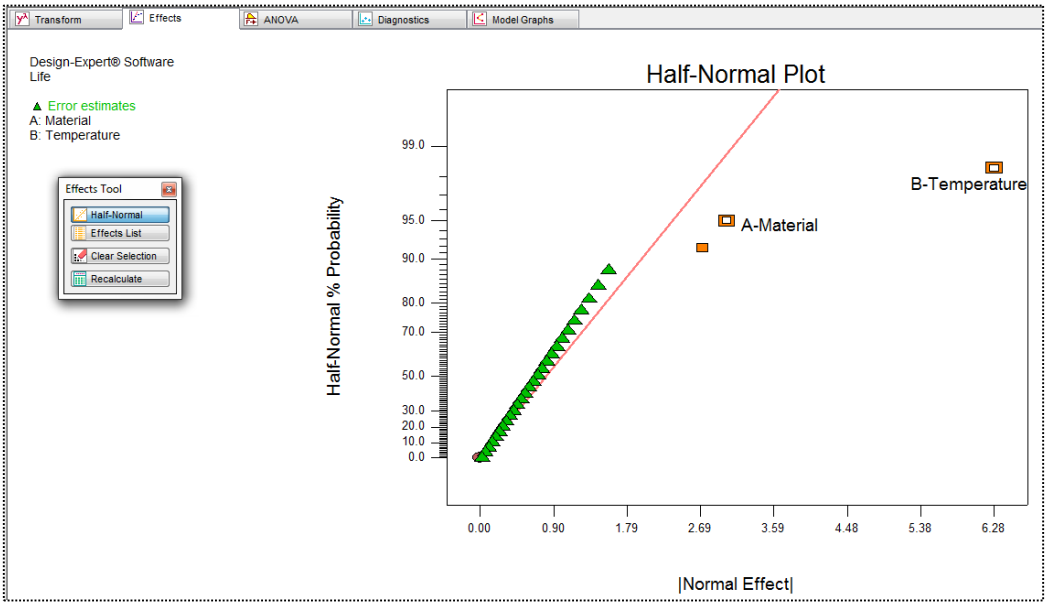
Then choose **Load an existing simulation** and click **Next>**. Finally, click the file named **Battery.simx** and **Open** it. If this file does not appear, choose Help, Tutorials>Find Tutorials Online and you will be directed to a website to download them. If you still can't find it, e-mail support@statease.com for help. You should now see data from the experiment. This is a good time to preserve your work: Select **File** and **Save As**. Change the file name to **Battery.dxpx** and **Save**.

Now go to the **Analysis** branch of the program and click the node labeled **R1:Life**. This brings up options for applying response transformations.



First step in the analysis – transformation options

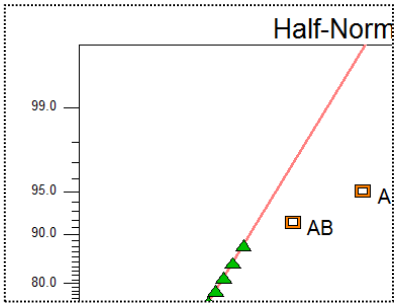
Leave the transformation at the default of "None" and go ahead and click the **Effects** button displayed next in the toolbar for response analysis. Design-Expert now provides an initial effect selection and displays it graphically on a specialized statistical plot called a "half-normal."



Initial effect selection

☒ ***Explore details on how the half-normal plot is constructed for general factorial designs***: The program displays the absolute value of all effects (plotted as squares) on the bottom axis. The procedure is detailed in a presentation by Patrick Whitcomb on "Graphical Selection of Effects in General Factorials" (2007 Fall Technical Conference co-sponsored by the American Society for Quality and the American Statistical Association) – contact Stat-Ease for a copy.

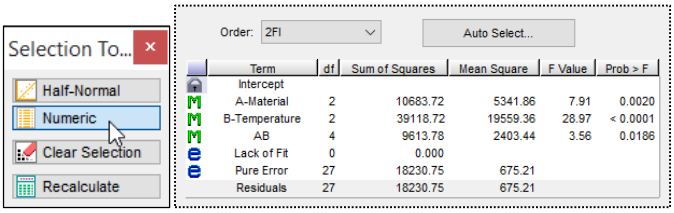
Design-Expert pre-selected two outstanding effects – the main effects of factors A and B. You can, and in this case should, modify the default effect selection. Move your mouse cursor over the unlabeled square and click it. (Note that this goes both ways, that is, you can deselect chosen effects with a simple mouse click.)



Another effect chosen

Interaction AB is now identified. Notice that Design-Expert adjusts the line to exclude the chosen effects. You will gain more practice on the use of half-normal plots for picking effects in the Two-Level Factorial Tutorial. It's best to now press ahead in this case.

☒ ***Explore the Numeric (Effects) List***: For statistical details, press the **Numeric** bar on the floating **Effects Tool**.

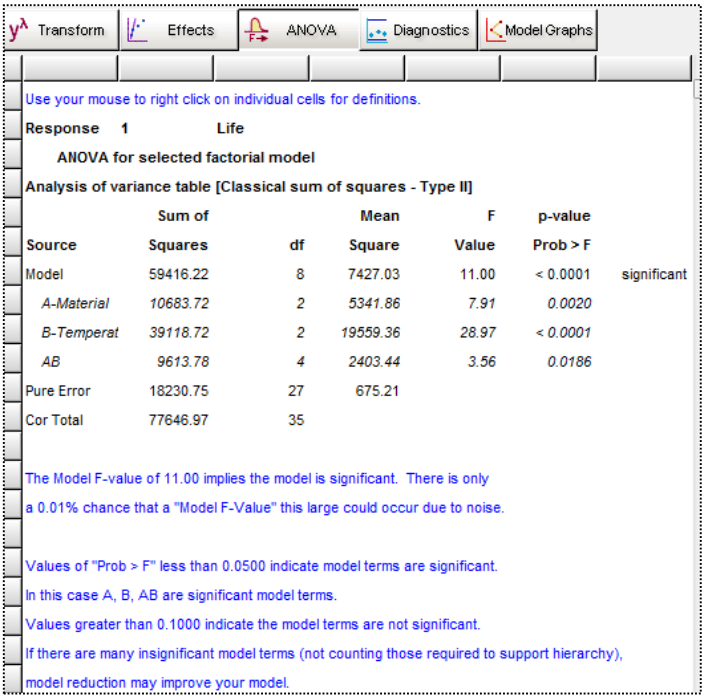


Numeric Selection showing Effects

Notice the designation "M" for the selected model terms A, B and AB and the "e" next to the pure error line in this statistical spreadsheet. You may be wondering why there are so many estimates of pure error. (If not, skip ahead!) Each subgroup of 4 replications provides 3 degrees of freedom ("df") of pure error. This was done for all 9 factor combinations (3x3) which yields 27 df (= 3*9) in total for estimating pure error.

This screen also provides many features for model selection, which will be covered in tutorials on response surface methods (RSM).

Click the **ANOVA** tab to see the analysis of variance for this chosen model. If you do not see annotations in blue text as shown below, select View, Annotated ANOVA.



Transform | Effects | ANOVA | Diagnostics | Model Graphs

Use your mouse to right click on individual cells for definitions.

Response 1 Life

ANOVA for selected factorial model

Analysis of variance table [Classical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	59416.22	8	7427.03	11.00	< 0.0001	significant
A-Material	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*	
B-Temperat	*39118.72*	*2*	*19559.36*	*28.97*	*< 0.0001*	
AB	*9613.78*	*4*	*2403.44*	*3.56*	*0.0186*	
Pure Error	18230.75	27	675.21			
Cor Total	77646.97	35				

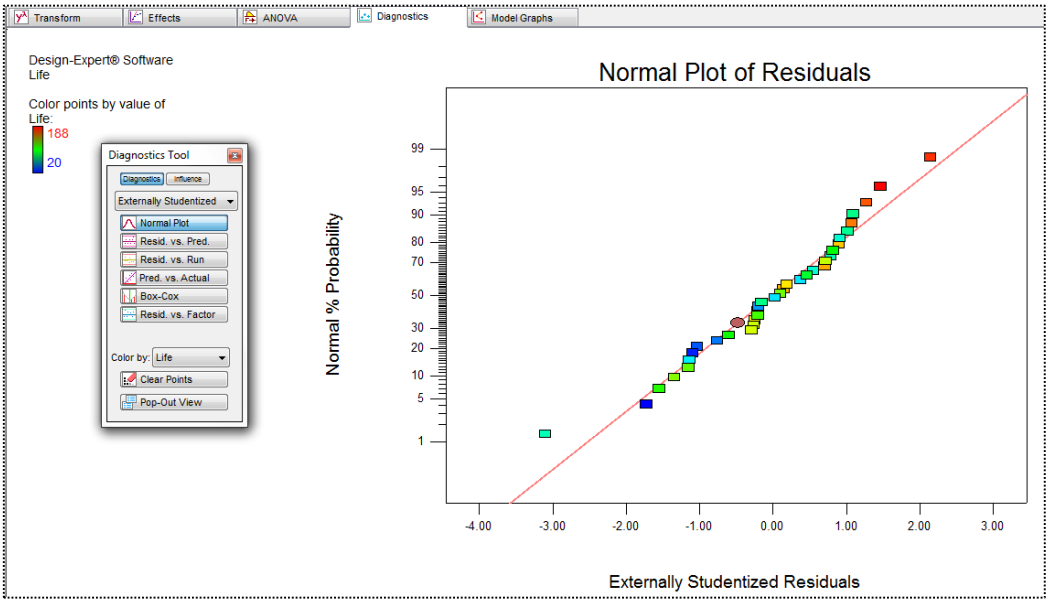
The Model F-value of 11.00 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.
In this case A, B, AB are significant model terms.
Values greater than 0.1000 indicate the model terms are not significant.
If there are many insignificant model terms (not counting those required to support hierarchy), model reduction may improve your model.

Annotated ANOVA Report

☒ ***Explore the other details provided under the ANOVA tab:*** Scroll down or press bookmarks on the floating tool to see post-ANOVA statistics such as R-Squared. As you can conclude for yourself by reading the annotations, the results look good. Further down the report are details of the model based on nominal contrasts. We provide a breakdown on this in the Experiment Design Made Easy workshop. To keep this tutorial moving, it's best not to get bogged down in the mathematics of modeling categorical factors, so press ahead.

Open the **Diagnostics** tab and examine the residual graphs. By default you see the normal plot of residuals, which ideally fall more-or-less in line. The pattern here is a bit askew but not badly abnormal, so do not worry.



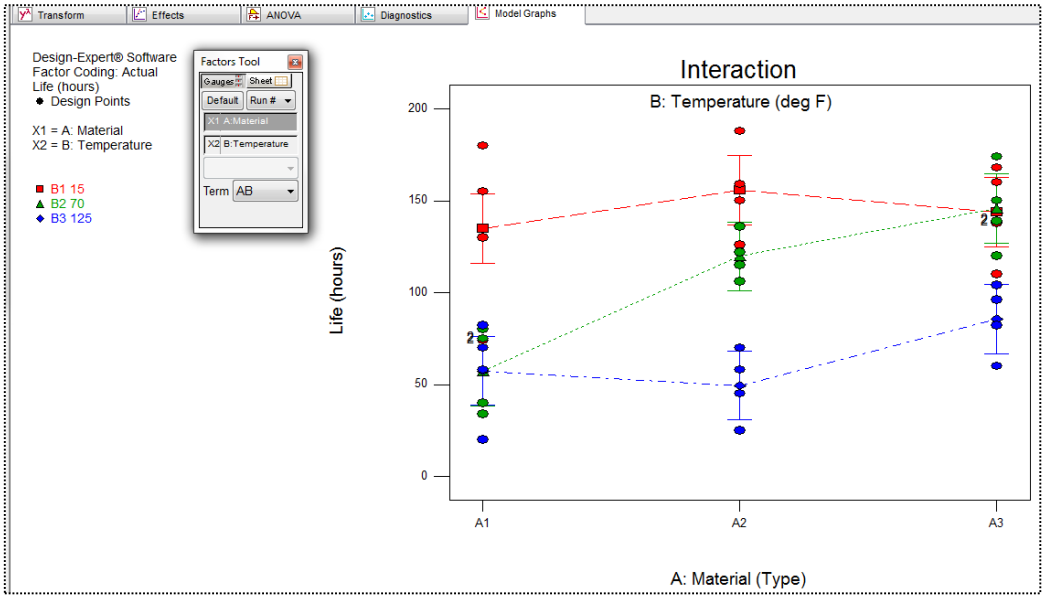
Normal plot of residuals – looks OK

☒ ***Explore all the other diagnostic plots:*** Take some time now to click your way down the floating tool and then over to the Influence side for viewing every one of the diagnostics plots and the final report.

The tedious, but necessary, model-fitting and statistical validation is now completed, so you are free and clear to finally assess the outcome of the experiment and decide whether any materials provide uniformly long battery-life regardless of temperature.

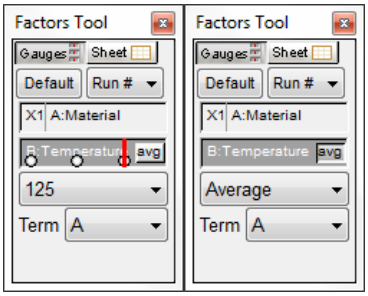
Present the Experimental Findings

Click the **Model Graphs** to view the long-awaited results. Design-Expert automatically presents the AB interaction plot – identified by the Term window on the floating Factors Tool.



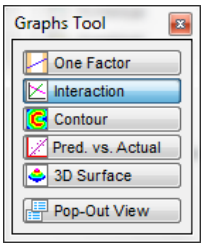
Default model graph – interaction plot with A on bottom (X1) axis

☒ ***Explore an over-simplified factor plot:*** Choose the One Factor plot view via the View menu, or by simply pressing the appropriate button on the floating Graphs Tool. Another way to bring up a one-factor plot (the main effect of A or B, in this case) is by clicking the down-list arrow (▼) for the Term selection on the Factors Tool. Try all these approaches if you like, but expect to be warned about presenting main effects of factors that interact. This can be very misleading. In this case it will be a mistake to look at either material or temperature effects alone, because one factor depends on the other. However, while you are at this, explore options on the **Factors Tool** for **B: Temperature**. Press the three buttons from left to right and notice how much the effects change due to the interaction. The press **avg**.



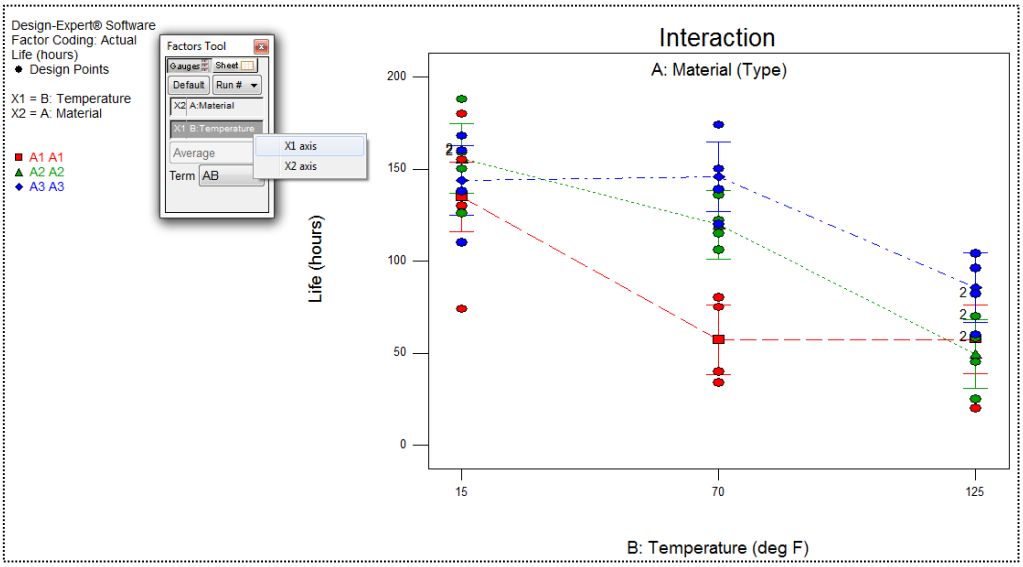
Viewing each Temperature and then its average for the effect plot of Material

Notice how the least significant difference (LSD) bars contract after the averaging. But nevertheless, this is not helpful because it obscures the interaction. On the **Graphs Tool** press the **Interaction** plot to bring back the true picture.



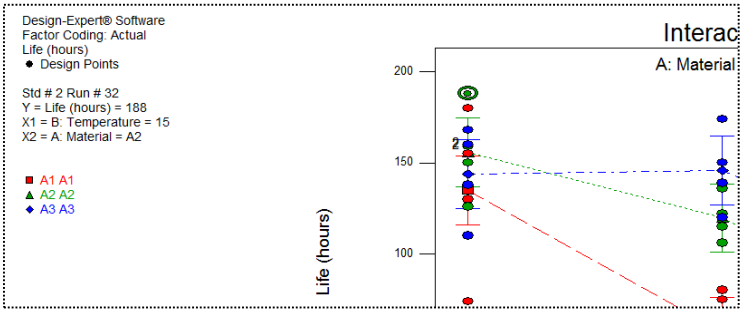
Graphs tool – interaction plot selected

Right- click the **Temperature** factor on the floating **Factors Tool** and change it to the **X1 Axis**, thus producing an interaction graph with the ordinal factor displayed in a continuous manner and the nominal factor (material) laid out discretely as separate lines. This makes it easier to interpret your results.



Effect graph with temperature on bottom axis

☒ ***Explore how the software identifies points:*** Click the highest point (green) at the upper left of the graph.



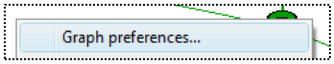
Point highlighted for identification

Note how to the left of the plot the software identifies the point by:

- Standard order number (2) and run number (randomized)
- Factor levels "X" (temperature of 15 with material A2).
- Actual response "Y" (188)

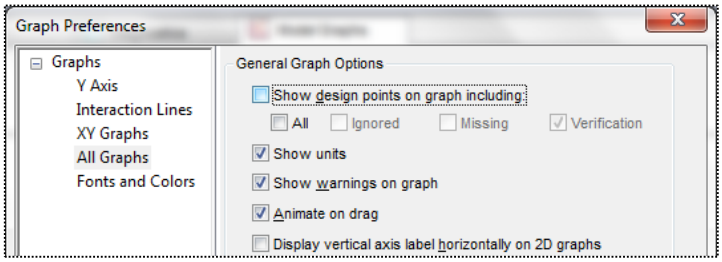
The actual results are represented by various-colored circles. If there are multiples, the program displays a number; in this case quite a few labeled "2". Click these points multiple times to see details on each and every one of them. You can also click on the non-circular symbols (square, triangle or diamond) to display the predicted outcome pairwise comparisons. Try this!

To produce a cleaner looking plot, go to **View** and deselect **Show Legend**. Now let's do some more clean-up for report purposes: Right-click over the graph and select **Graph Preferences**.



Right-click menu selection for graph preferences

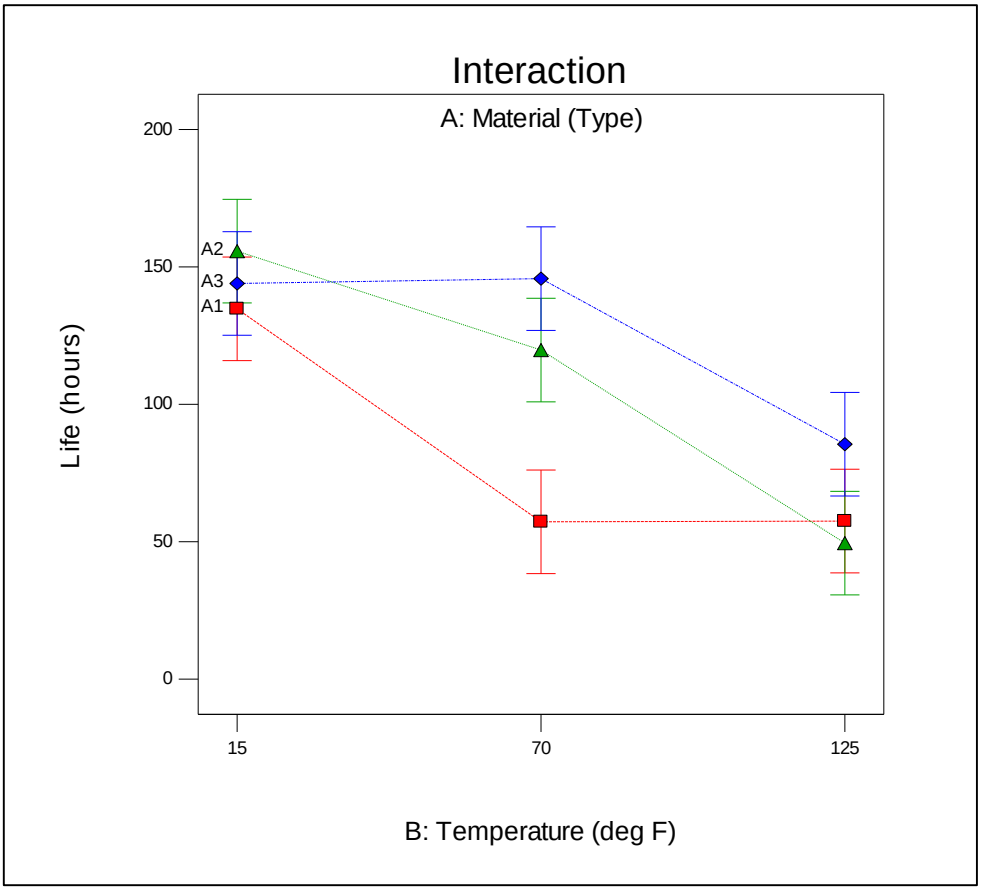
Now click the **All Graphs** tab and turn off (uncheck) the **Show design points on graph** option, as shown below.



Turning off design points

Press **OK**.

☒ ***Explore copy/past to Microsoft Word (or the like):*** This is an optional sidetrack on this tutorial: To have your graph look like that shown below for reporting purposes, do the following: **Edit**, **Copy** from Design-Expert, then **Paste** into Microsoft Word.

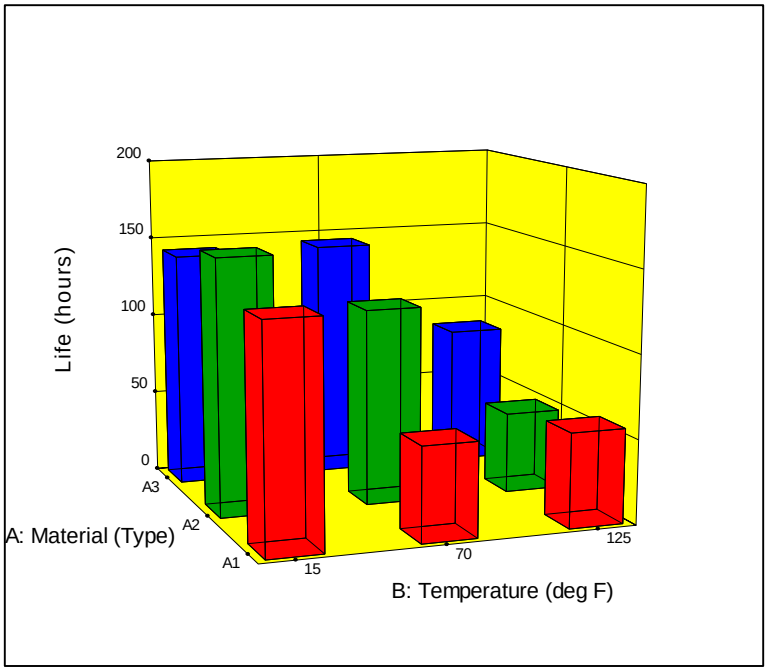


Clean-looking interaction graph

From this graph you can see that all three materials work very well at the low temperature (15 degrees). Based on the overlapping LSD bars, it would be fair to say that no material stands out at this low temperature end of the scale. However, the A1 material clearly falls off at the 70 degree temperature, which would be encountered most often, so it must be rejected. None of the materials perform very well at the highest temperature (125 degrees), but the upper end of the LSD bar for A2 barely overlaps the bottom end of the LSD bar for A3. Therefore, with respect to temperature sensitivity, material A3 may be the most robust material for making batteries.

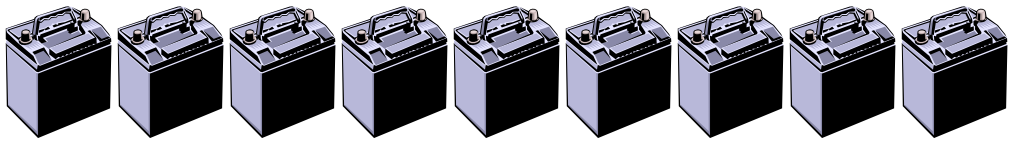
Finally, if you do have an opportunity to present graphics in color, here's a dazzling new and easy way to display general factorial effects with Design-Expert: Click **3D Surface** on the floating **Graphs Tool**.

Now place your mouse cursor on the graph – notice that it changes to a hand (✋). While pressing the left mouse button, spin the graph so the temperature axis is at the bottom. (Alternatively, to match our graph most precisely, select View, Show Rotation and enter coordinates of h (horizontal) 20 and v (vertical) 80.)



3D surface plot – rotated slightly for a better view

The 3D view presents a different perspective of the general factorial effects – more on a macro level of the overall experimental landscape. Now the inferiority of material A1 (red bars) becomes obvious: The other two materials tower over it at the mid-temperature of 70 degrees F. Clearly the next step is to eliminate material A1 from contention and perhaps do some further investigations on A2 and A3.



General Factorial Tutorial
(Part 2 – Making Factors Numeric)

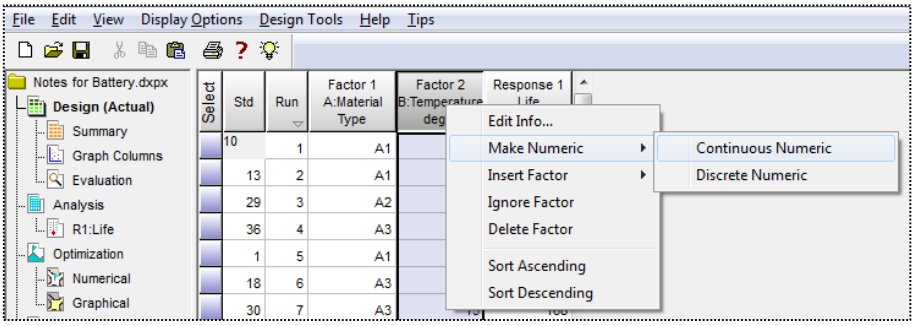
Continued – A Case Study on Battery Life

In the preceding, General Factorial Tutorial – Part 1, you treated all factors categorically. The main purpose of Part 2 of this tutorial is to illustrate some of the functions built into Design-Expert that can be used to make effects graphs better fit the type of data we're dealing with, in addition to making the results easier to interpret and understand. (We've already said that Montgomery's classic battery experiment could have been handled by using the Response Surface tab in Design-Expert software and constructing a one-factor design on temperature, with the addition of one categorical factor at three levels for the material type. Never fear, we'll cover response surface methods like this from the ground up later in the Response Surface Tutorials.) But to demonstrate the flexibility of Design-Expert, let's see how you can make the shift from categoric to numeric after-the-fact, even within the context of a general factorial design.

To get started, re-open the file named **Battery.dxpx** saved earlier. If the file is already open, then click the **Design** node.

Changing a Factor from Categoric to Numeric

Place your cursor on the **B:Temperature** factor column heading, right click, and move your cursor to **Make Numeric** and then **Continuous Numeric**. (The "discrete" option could also work in this case. It works well for numeric factors that for convenience-sake must be set at specific levels. For example, imagine that the testing chamber for the batteries has only three settings – 15, 70 and 125.)

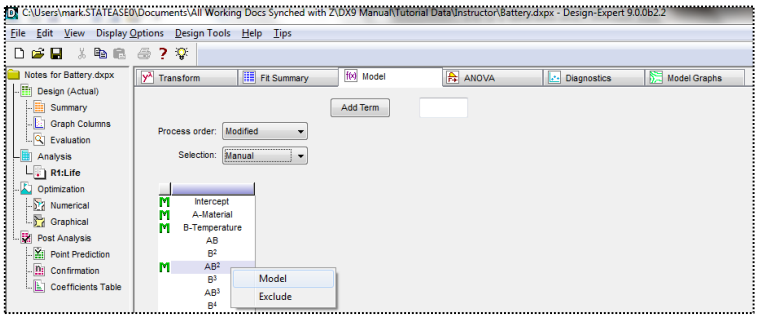


Options for editing a factor

The software pops up a warning not to do this if the factor really is categorical. To acknowledge it and move on, click the **OK** button below the message.

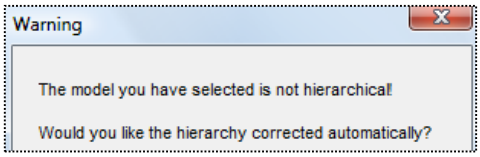
Re-Analyzing the Results

To re-analyze your data, click the analysis node labeled **Life**. Then click the **Model** tab. When you designated a factor being numeric, Design-Expert automatically shifted to fitting a polynomial, such as those used for response surface methods. To model non-linear response to temperature (factor B), double-click the **AB^2** term, or via a right click add it to the **Model** as shown below. (Squared terms capture curvature.)



Model selection screen

Click the **ANOVA** tab. You will get a warning about hierarchy.



Hierarchy warning

This warning arises because you chose a higher order term without support by parent terms, in this case: AB and B^2. Click **Yes** and move on.

☒ ***Explore statistical reasons for maintaining model hierarchy:*** For details, search out the topic on "Model Hierarchy Check" in the Help System.

The ANOVA report now displays in the view (annotated or not) that you used last. By comparing this output with the ANOVA done in Part One, observe that the lines for the model and residual come out the same, but the terms involving B differ. In Part One we treated factor B (temperature) categorically, although in an ordinal manner. Now that this factor is recognized explicitly as numeric, what was the effect of B is now broken down to two model terms – B and B^2, and AB becomes AB plus AB^2.

Transform | Fit Summary | Model | ANOVA | Diagnostics | Model Graphs

Use your mouse to right click on individual cells for definitions.

Response 1 Life

Hierarchical terms added after manual regression

AB, B^2

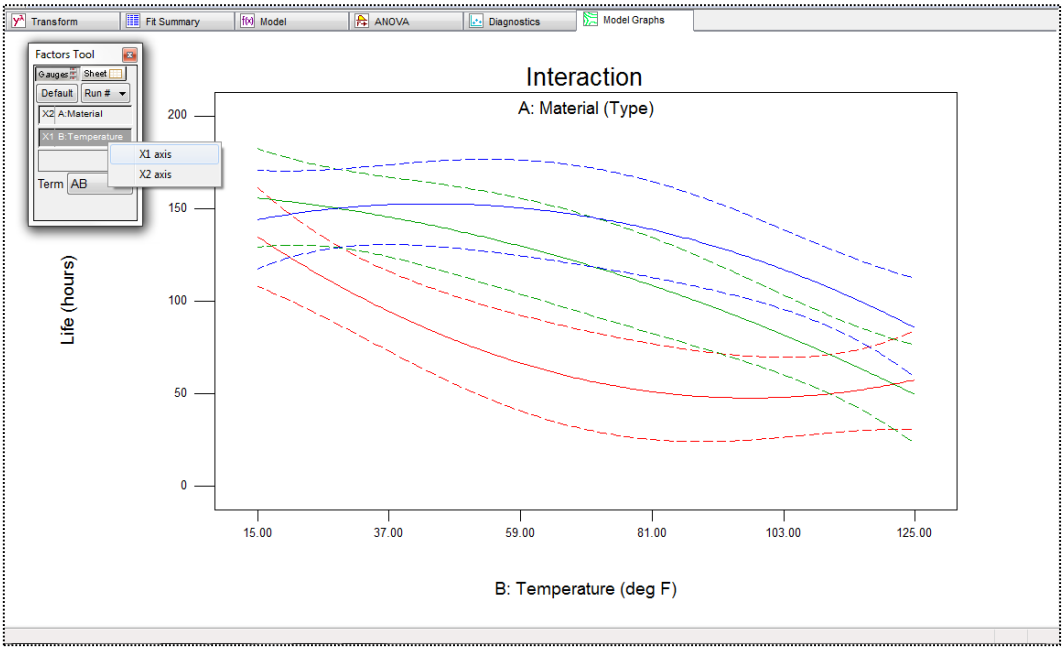
ANOVA for Response Surface Reduced Cubic model

Analysis of variance table [Classical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	59416.22	8	7427.03	11.00	< 0.0001	significant
A-Material	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*	
B-Temperatu	*39042.67*	*1*	*39042.67*	*57.82*	*< 0.0001*	
AB	*2315.08*	*2*	*1157.54*	*1.71*	*0.1991*	
B^2	*76.06*	*1*	*76.06*	*0.11*	*0.7398*	
AB^2	*7298.69*	*2*	*3649.35*	*5.40*	*0.0106*	
Pure Error	18230.75	27	675.21			
Cor Total	77646.97	35				

ANOVA output

The whole purpose of this exercise is to make a better looking effects graph. Let's see what this looks like by clicking the **Model Graphs** tab. Go to the floating **Factors Tool**, right-click **B** (Temperature) and change it to the **X1 Axis**.

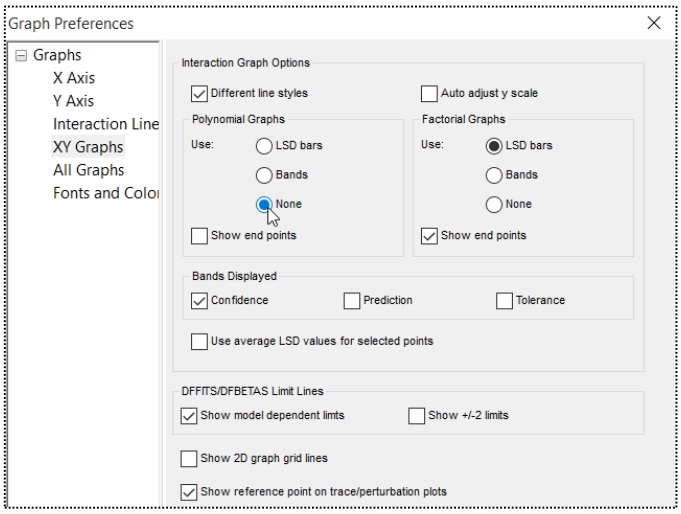


Viewing the interaction with temperature on X-axis

You now have a plot characterizing the data from Part One of this ongoing case study, except that the above lines are now continuous with temperature, whereas in Part One they were displayed as discrete (categorical) segments. Notice that the curves by temperature (modeled by B^2) depend on the type of material (A). This provides graphical verification of the significance of the AB^2 term in the model.

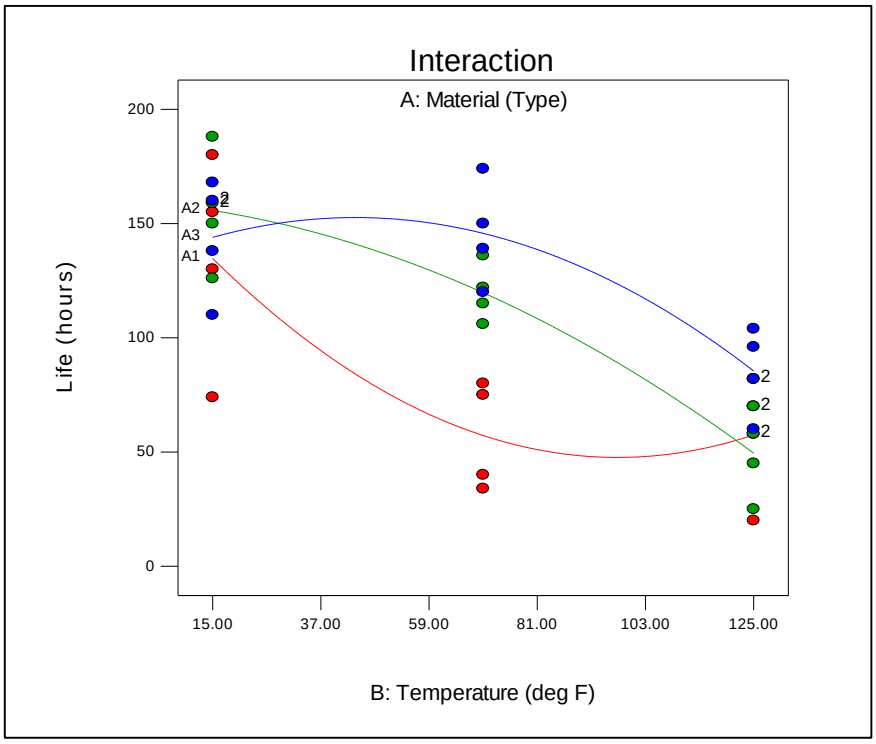
The dotted lines are the confidence bands, which in this case cause more trouble than they're worth for conveying the uncertainty in fitting, so let's change Graph Preferences to take these off. First, right-click over the graph and select **Graph**

Preferences. Now click the **XY Graphs** tab and in the **Polynomial Graphs** section click the **None** option as shown below.



Turning off confidence bands

Press **OK**. That cleans up the picture, but perhaps too much! Let's put the points back on for perspective. Again, right-click over the graph and select **Graph Preferences**. Now click the **All Graphs** tab and turn on (check) the **Show design points on graph** option. Your plot should now look like that copied out below. (Note: if you don't have the A1, A2, and A3 labels, remove the legend from your graph by going to View->Show legend).

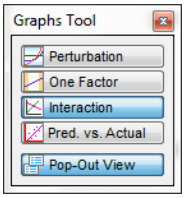


Presentation of battery life made of various materials versus temperature on continuous scale

The conclusions remain the same as before: Material A3 will maximize battery life with minimum variation in ambient temperature. However, by treating temperature numerically, predictions can be made at values between those tested. Of course, these findings are subject to confirmation tests.

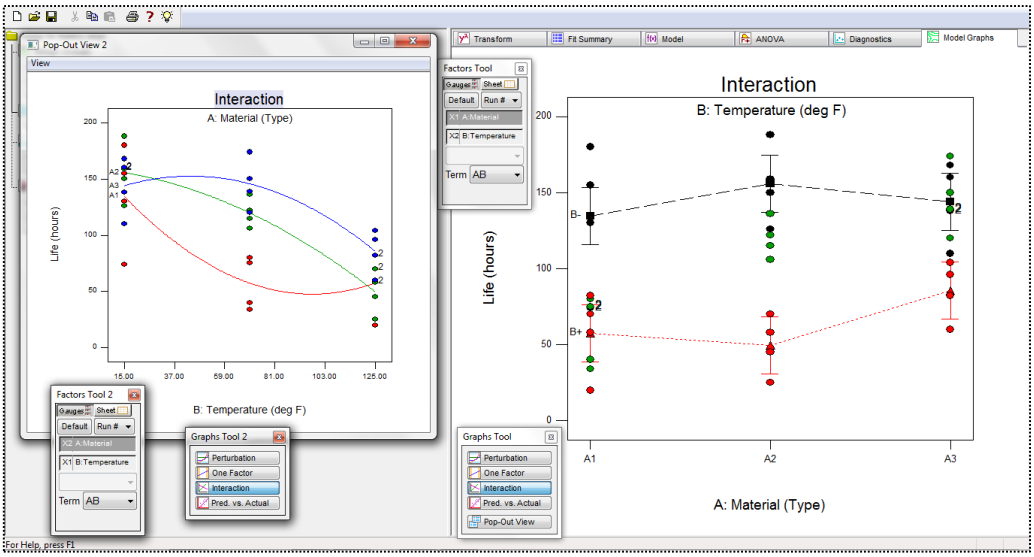
Postscript: Demo of "Pop out" View

Before exiting Design-Expert, give this a try: Press the **Pop out View** on the floating **Graphs Tool** (or go to the View menu and select Pop out View).



Pop out View

This pushes the current graph out of its fixed Windows pane into a 'clone' that floats around on your screen. Now on the original **Factors Tool** right click on **A** (Material) and return it to the **X1 Axis**. Then do an Alt-Tab to bring back the clone of the previous view back on your current window.



Two ways of viewing the battery life results

You can present Design-Expert outputs both ways for your audience:

- Curves for each material as a function of temperature on the X1 axis, or
- Two temperature lines connected to the three discrete materials as X1.

Another way to capture alternative graphs is to copy and paste them into a word-processor, spreadsheet, or presentation program. Then you can add annotations and explanations for reporting purposes.

